

# 9 Statistical Measures

9.1 Introduction to Statistics

9.2 Mean

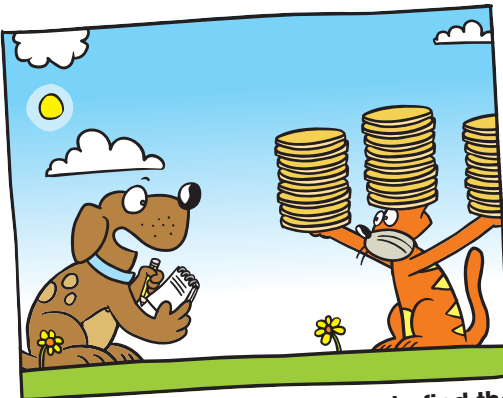
9.3 Measures of Center

9.4 Measures of Variation

9.5 Mean Absolute Deviation



"Mom, my owner, and Fluffy have agreed to participate in my survey. Will you be my fourth participant?"



"Please hold still. I am trying to find the mean of 6, 8, and 10 by dividing their sum into three equal piles."

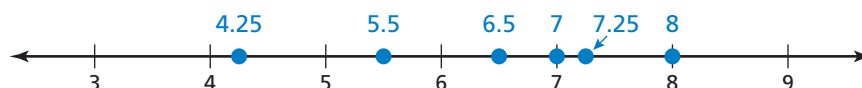


Help! I've got an itch. Can someone please scratch me?

# What You Learned Before

## Ordering Decimals (4.NF.7)

**Example 1** Use a number line to order 8, 6.5, 7.25, 5.5, 4.25, and 7 from least to greatest.



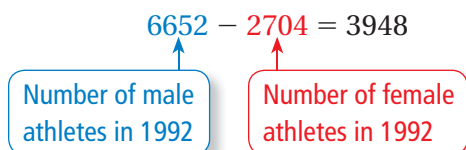
### Try It Yourself

Use a number line to order the numbers from least to greatest.

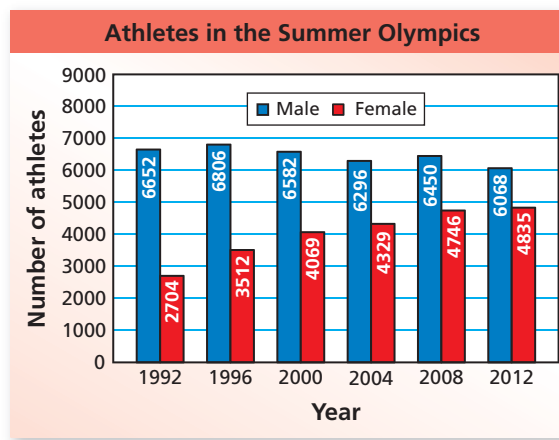
- 7.25, 4.5, 6.5, 6, 5.5, 8.75
- 4, 2.5, 3.25, 5.5, 4.5, 6.75
- 6.25, 3, 2.5, 3.5, 5.75, 5
- 1.25, 5.5, 4.75, 4.5, 3.5, 2.25

## Analyzing Double Bar Graphs (3.MD.3, 4.NBT.4)

**Example 2** How many more male athletes than female athletes participated in the 1992 Summer Olympics?



3948 more male athletes participated.



**Example 3** How many athletes participated in the 2000 Summer Olympics?

$6582 + 4069 = 10,651$  athletes participated in the 2000 Summer Olympics.

### Try It Yourself

- How many more female athletes participated in the 2012 Summer Olympics than in the 1992 Summer Olympics?
- Describe the relationship between the number of athletes in the 2000 Summer Olympics and the number of athletes in the 2004 Summer Olympics.





## 9.1 Introduction to Statistics

**Essential Question** How can you tell whether a question is a statistical question?

Your heart rate is the number of times your heart beats in a certain time period, such as 1 minute. To measure your heart rate, you can check your pulse. The illustration shows how to check your pulse by pressing lightly on your wrist.



Here are other places to check your pulse:

- inside your elbow
- side of your neck
- top of your foot

### 1 ACTIVITY: Using Data to Answer Questions

**Work with a partner.**

- a. Find your pulse by counting the number of beats in 10 seconds. Have your partner keep track of the time. Write a rate to describe your result.

- b. Complete the ratio table. What is your heart rate in beats per minute?

Time (seconds)	10	30	60
Number of Beats			

- c. Collect the recorded heart rates (in beats per minute) of the students in your class, including yourself. Compare the heart rates.

- d. **MODELING** Make a *line plot* of your data. Then answer the following questions:

- How many values are in your data set?
- Do the heart rates *cluster* around a particular value or values?
- Are there any *peaks* or *gaps* in the data?
- Are there any unusual heart rates that are far removed from the other values?

- e. **REASONING** How would you answer the following question by using only one value? Explain your reasoning.

*“What is the heart rate of sixth grade students?”*

- f. **REASONING** Read and compare the following questions. How did you answer each question? Could the answer be the same for both questions? Explain.

- *What is your heart rate?*
- *What is the heart rate of sixth grade students?*



#### Statistics

In this lesson, you will

- recognize statistical questions.
- use dot plots to display numerical data.

Learning Standards

6.SP.1

6.SP.4

## 2 ACTIVITY: Identifying Types of Questions

Work with a partner.

- a. Answer each question below on your own. Then compare your answers with your partner's answers. For which questions should your answers be the same? For which questions might your answers be different?
  1. What is your shoe size?
  2. How many states are in the United States?
  3. How many brothers and sisters do you have?
  4. How many U.S. presidents have been in office?
  5. What is your favorite type of movie?
  6. How tall are you?
- b. **CONJECTURE** Some of the questions above are considered *statistical* questions. Which ones do you think they are? Why?

### Math Practice

# 3

#### Build Arguments

How can comparing your answers help you support your conjecture?

## 3 ACTIVITY: Analyzing a Question in a Survey

Work with a partner. A student asks the following question in a survey:

*“Do you prefer salty potato chips or healthy granola bars to be sold in the school’s vending machines?”*



- a. Do you think this is a fair question to ask in a survey? Explain.
- b. **LOGIC** Identify the words in the question that may influence someone's response. Then explain how you can reword the question.
- c. How might the results of the survey differ when the student asks the original question and your reworded question in part (b)?

## What Is Your Answer?

4. **REASONING** What do you think “statistics” means?
5. **IN YOUR OWN WORDS** How can you tell whether a question is a statistical question? Give examples to support your explanation.
6. Find the least and the greatest heart rates in your class. How can you use these two values to answer the question in Activity 1(e)?
7. Create a one-question survey. Explain why your question is a statistical question. Then conduct your survey and organize your results in a line plot. Make three observations about your data set.

### Practice

Use what you learned about different types of questions to complete Exercises 4–7 on page 394.



## 9.1 Lesson

### Key Vocabulary

statistics, p. 392  
statistical question,  
p. 392

**Statistics** is the science of collecting, organizing, analyzing, and interpreting data. A **statistical question** is one for which you do not expect to get a single answer. Instead, you expect a variety of answers, and you are interested in the distribution and tendency of those answers.

Recall that a dot plot uses a number line to show the number of times each value in a data set occurs. Dot plots show the *spread* and the *distribution* of a data set.

### EXAMPLE 1 Answering a Statistical Question

You conduct a science experiment on house mice. Your teacher asks you, “What is the weight of a mouse?”

a. Is this a statistical question? Explain.

- Because you can anticipate that the weights of mice will vary, it is a statistical question.

b. You weigh some mice and record the weights (in grams) in the table. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

Draw a number line that includes the least value, 18, and greatest value, 28. Then place a dot above the number line for each data value.

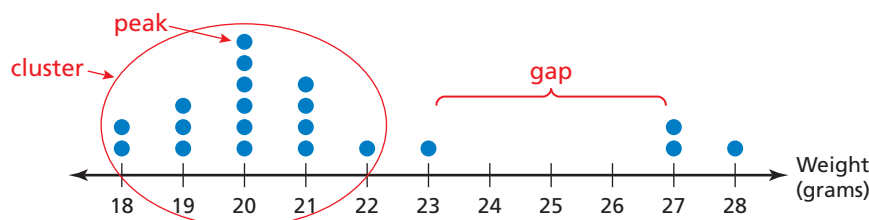


#### Weights (grams)

20	19	21	20
18	20	27	21
28	23	20	19
20	21	18	27
19	22	21	20

### Study Tip

Dot plots are sometimes called *line plots*. It is easy to see clusters, peaks, and gaps in a dot plot.



Most of the data are clustered around 20. There is a peak at 20 and a gap between 23 and 27.

c. Use the distribution of the data to answer the question.

- Most mice weigh about 20 grams.

### On Your Own

Now You're Ready  
Exercises 8–16

1. The table shows the ages of some people who retired early. You are asked, “How old are people who retire early?”
  - a. Is this a statistical question? Explain.
  - b. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
  - c. Use the distribution of the data to answer the question.

Ages			
60	61	59	60
62	56	64	59
58	60	61	60
59	60	58	61

## EXAMPLE 2 Using a Dot Plot

You record the high temperature every day while at summer camp in August. Then you create the vertical dot plot.

- a. How many weeks were you at summer camp?

Because there are 28 data values on the dot plot, you were at camp 28 days.

$$28 \text{ days} \cdot \frac{1 \text{ week}}{7 \text{ days}} = 4 \text{ weeks}$$

- So, you were at summer camp for 4 weeks.

- b. How can you collect these data? What are the units?

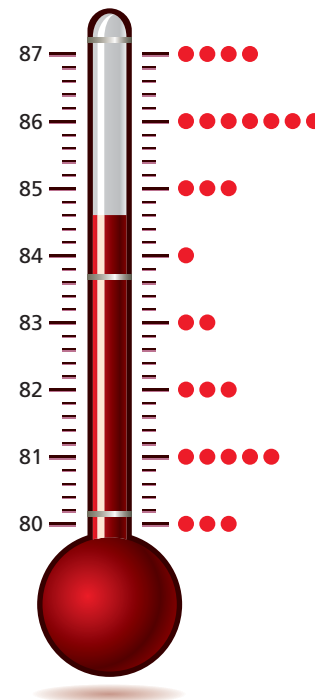
- You can collect these data with a thermometer. The units are degrees Fahrenheit ( $^{\circ}\text{F}$ ).

- c. Write a statistical question that you can answer using the dot plot. Then answer the question.

One possible statistical question is:

*What is the daily high temperature in August?*

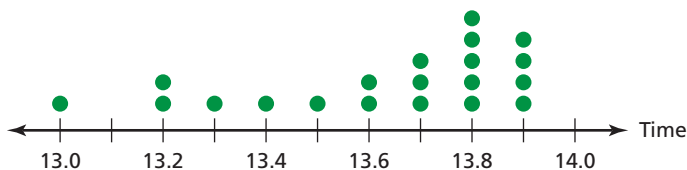
- The high temperatures are spread out with about half of the temperatures around  $81^{\circ}\text{F}$  and half of the temperatures around  $86^{\circ}\text{F}$ .



## On Your Own

**Now You're Ready**  
Exercises 17 and 18

2. The dot plot shows the times of sixth grade students in a 100-meter race.



- How many students ran in the race?
- How can you collect these data? What are the units?
- Write a statistical question that you can answer using the dot plot. Then answer the question.



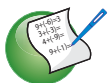
## 9.1 Exercises



### Vocabulary and Concept Check

- VOCABULARY** What is a statistical question? Give an example.
- CRITICAL THINKING** What process can you use to answer a statistical question?
- NUMBER SENSE** The results of a survey are shown in the table. Did the survey ask a statistical question? Explain.

Miles			
6	1	9	2
2	5	4	9
8	10	6	6
5	1	8	1



### Practice and Problem Solving

Answer the question. Tell whether your answer would be the same as your classmates'.

- How many inches are in 1 foot?
- How many pets do you have?
- On what day of the month were you born?
- How many senators are in Congress?

Determine whether the question is a statistical question. Explain.

- What is the eye color of sixth grade students?
- At what temperature (in degrees Fahrenheit) does water freeze?
- How many pages are in the favorite books of students your age?
- How many hours do sixth grade students use the Internet each week?

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

12.

Number of Fouls					
2	1	2	0	0	2
2	1	6	1	1	0

13.

Camper Registrations				
21	25	25	22	21
23	24	26	25	16
24	26	22	25	22

14.

Years			
2011	2008	2013	2009
2009	2010	2010	2009
2010	2012	2009	2010

15.

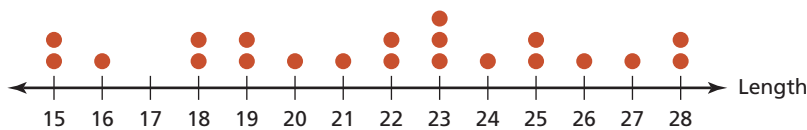
Test Scores				
85	80	83	90	88
82	83	81	80	89
89	84	86	87	83

Hours of Homework			
2	4	3	2
1	2	2	1
2	3	5	2

16. **SURVEY** You conduct a survey to answer: "How many hours does a sixth grade student spend on homework during a school night?" The table shows the results.
- Is this a statistical question? Explain.
  - Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
  - Use the distribution of the data to answer the question.



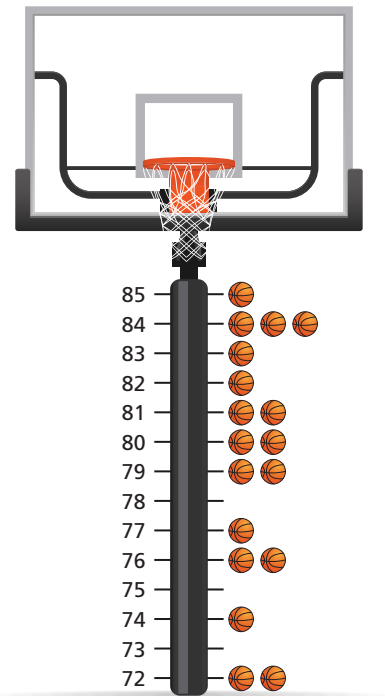
- 2 17. **EARTHWORMS** The dot plot shows the lengths of earthworms.



- How many earthworms does it represent?
- How can you collect these data? What are the units?
- Write a statistical question that you can answer using the dot plot. Then answer the question.

18. **BASKETBALL** The vertical dot plot shows the heights of the players on a recent NBA championship team.

- How many players were on the team?
- How can you collect these data? What are the units?
- Write a statistical question that you can answer using the dot plot. Then answer the question.

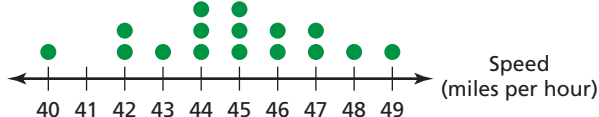


Use the Internet to research and identify the method of measurement and the units used when collecting data about the topic.

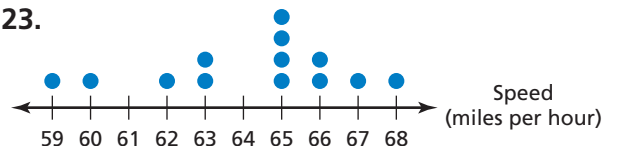
19. wind speed                      20. amount of rainfall                      21. earthquake intensity

The dot plot shows the speeds of cars in a traffic study. Estimate the speed limit. Explain your reasoning.

22.



23.



24. **REASONING** “How many letters are in the English alphabet?” is *not* a statistical question. Write a question about letters that is a statistical question. Explain your reasoning.

25. **Reasoning** A bar graph shows the favorite colors of 30 people. Does it make sense to describe the distribution of these data? Explain.



## Fair Game Review what you learned in previous grades & lessons

Tell whether the ordered pair is a solution of the equation. (Section 7.4)

26.  $y = 4x$ ; (2, 8)                      27.  $y = 3x + 5$ ; (3, 15)                      28.  $y = 6x - 15$ ; (4, 9)

29. **MULTIPLE CHOICE** A point is reflected in the  $x$ -axis. The reflected point is (4, -3). What is the original point? (Section 6.5)

- (A) (-3, 4)                      (B) (-4, 3)                      (C) (-4, -3)                      (D) (4, 3)

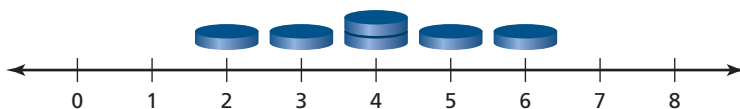
## 9.2 Mean

**Essential Question** How can you find an average value of a data set?

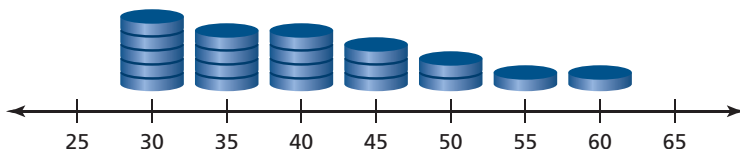
### 1 ACTIVITY: Finding a Balance Point

Work with a partner. Discuss the distribution of the data. Where on the number line do you think the data set is *balanced*? Is this a good representation of the average? Explain.

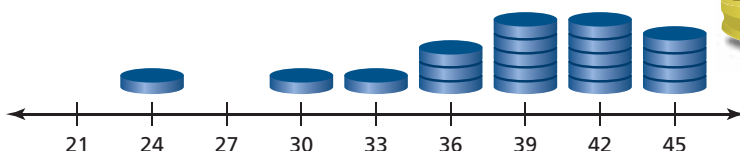
- a. number of quarters brought to a batting cage



- b. annual income of recent graduates  
(in thousands of dollars)



- c. hybrid fuel economy (miles per gallon)



### 2 ACTIVITY: Finding a Fair Share

Work with a partner. It costs \$0.25 to hit 12 baseballs in a batting cage. The table shows the numbers of quarters six friends bring to the batting cage. They want to group the quarters so that everyone has the same amount.

Quarters					
John	Lisa	Miguel	Matt	Cheryl	Jean
6	3	4	5	2	4

Use counters to represent each number in the table. How can you use the counters to determine how many times each friend can use the batting cage? Explain how this procedure results in a "fair share."



COMMON  
CORE

#### Statistics

In this lesson, you will

- understand the concept of the mean of data sets.
- find the mean of data sets.
- compare and interpret the means of data sets.

#### Learning Standards

- 6.SP.2
- 6.SP.3
- 6.SP.5a
- 6.SP.5c

### 3 ACTIVITY: Finding an Average

Work with a partner. Use the information in Activity 2.

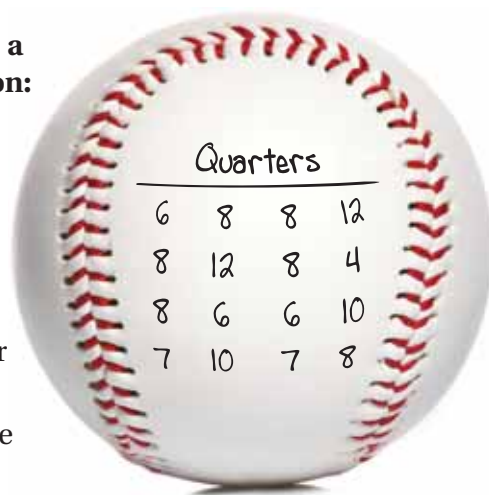
- What is the total number of quarters the group of friends brought to the batting cage?
- REASONING** How can you use math to find the average number of quarters that each friend brought to the batting cage? Find the average number of quarters. Why do you think this average represents a fair share?

### 4 ACTIVITY: Answering a Statistical Question

Work with a partner. The table shows the numbers of quarters several people bring to a batting cage. You want to answer the question:

*“How many quarters do people bring to a batting cage?”*

- Explain why this question is a statistical question.
- MODELING** Make a dot plot of the data. Use the distribution of the data to answer the question. Explain your reasoning.
- REASONING** Use an average to answer the question. Explain your reasoning.



### Math Practice 6

#### Use Clear Definitions

What does it mean for data to have an average? How does this help you answer the question?

## What Is Your Answer?

- IN YOUR OWN WORDS** How can you find an average value of a data set?
- Give two real-life examples of averages.
- Explain what it means to say the average of a data set is the point on a number line where the data set is balanced.
- There are 5 students in the cartoon. Four of the students are 66 inches tall. One is 96 inches tall.
  - How do you think the students decided that their average height is 6 feet?
  - Does a height of 6 feet seem like a good representation of the average height of the 5 students? Explain why or why not.



“Yup, the average height in our class is 6 feet.”

### Practice

Use what you learned about averages to complete Exercises 4 and 5 on page 400.



## Key Vocabulary

mean, p. 398  
outlier, p. 399

A mean is a type of average.

## Key Idea

### Mean

**Words** The **mean** of a data set is the sum of the data divided by the number of data values.

**Numbers** Data: 8, 5, 6, 9    Mean:  $\frac{8 + 5 + 6 + 9}{4} = \frac{28}{4} = 7$   
 4 data values

## EXAMPLE 1 Finding the Mean

### Text Messages Sent

Mark: 120  
Laura: 95  
Stacy: 101  
Josh: 125  
Kevin: 82  
Maria: 108  
Manny: 90

The table shows the number of text messages sent by a group of friends over 1 week. What is the mean number of messages sent?

- (A) 100      (B) 102      (C) 103      (D) 104

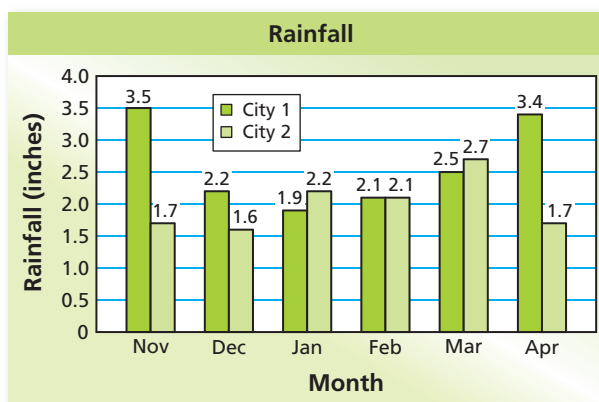
$$\begin{aligned} \text{mean} &= \frac{120 + 95 + 101 + 125 + 82 + 108 + 90}{7} \\ &= \frac{721}{7}, \text{ or } 103 \end{aligned}$$

Sum of the data  
Number of values  
Simplify.

The mean number of text messages sent is 103. The correct answer is (C).

## EXAMPLE 2 Comparing Means

The double bar graph shows the monthly rainfall amounts for two cities over a six-month period. Compare the mean monthly rainfalls.



City 1 mean:  $\frac{3.5 + 2.2 + 1.9 + 2.1 + 2.5 + 3.4}{6} = \frac{15.6}{6}, \text{ or } 2.6$

City 2 mean:  $\frac{1.7 + 1.6 + 2.2 + 2.1 + 2.7 + 1.7}{6} = \frac{12}{6}, \text{ or } 2$

Because 2.6 is greater than 2, City 1 averaged more rainfall.

### On Your Own

Find the mean of the data.

- 49, 62, 52, 54, 61, 70, 55, 53
- 7.2, 8.5, 7.0, 8.1, 6.7

An **outlier** is a data value that is much greater or much less than the other values. When included in a data set, it can affect the mean.

### EXAMPLE 3 Finding the Mean With and Without an Outlier

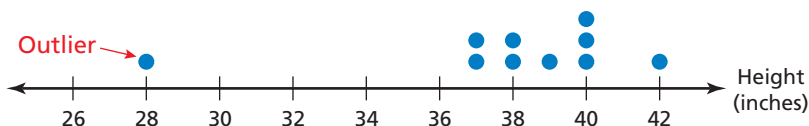
Shetland Pony Heights (inches)

40	37	39	40	42
38	38	37	28	40

The table shows the heights of several Shetland ponies.

- Identify the outlier.
- Find the mean with and without the outlier.
- Describe how the outlier affects the mean.

- Display the data in a dot plot.



The height of 28 inches is much less than the other heights. So, it is an outlier.

- Mean with outlier:

$$\frac{40 + 37 + 39 + 40 + 42 + 38 + 38 + 37 + 28 + 40}{10} = \frac{379}{10}, \text{ or } 37.9$$

Mean without outlier:

$$\frac{40 + 37 + 39 + 40 + 42 + 38 + 38 + 37 + 40}{9} = \frac{351}{9}, \text{ or } 39$$

- With the outlier, the mean is less than all but three of the heights. Without the outlier, the mean better represents the heights.

### On Your Own

For each data set, identify the outlier. Then describe how it affects the mean.

- Weights (in pounds) of dogs at a kennel:  
48, 50, 55, 60, 8, 37, 50
- Prices for flights from Miami, Florida, to San Juan, Puerto Rico:  
\$456, \$512, \$516, \$900, \$436, \$516

## 9.2 Exercises



### Vocabulary and Concept Check

1. **VOCABULARY** Arrange the words to explain how to find a mean.

the data values

divide by

the number of data values

add

then

2. **NUMBER SENSE** Is the mean always equal to a value in the data set? Explain.
3. **REASONING** Can you use the mean to answer a statistical question? Explain.



### Practice and Problem Solving

Describe an average value of the data.

4. Ages in a class: 11, 12, 12, 12, 12, 12, 13
5. Movies seen this week: 0, 0, 0, 1, 1, 2, 3

Find the mean of the data.

1

6.

Pets Owned	
Brandon	I
Jill	III
Mark	II
Nicole	IIII
Steve	0

7.

Brothers and Sisters	
Amanda	♀
Eve	♀ ♀ ♀ ♀ ♀
Joseph	♂ ♂ ♂ ♂
Michael	♂ ♂

8.

Sit-ups		
108	85	94
103	112	115
98	119	126
105	82	89

9.



10. **GOLF** The table shows tournament finishes for a golfer.

- a. What was the golfer's mean finish?
- b. Identify two outliers for the data.

Tournament Finishes							
1	1	2	1	1	12	6	2
15	37	1	2	1	26	9	1

Time (minutes)				
4.2	3.5	4.55	2.75	2.25

11. **COMMERCIALS** You and your friends are watching a television show. One of your friends asks, "How long are the commercial breaks during this show?"

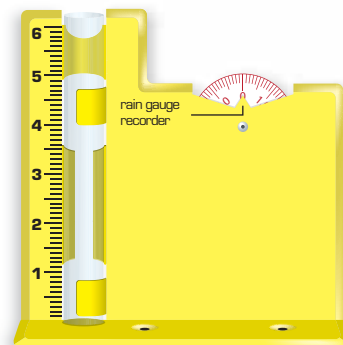
- a. Is this a statistical question? Explain.
- b. Use the mean of the values in the table to answer the question.



Month	Rainfall (inches)	Month	Rainfall (inches)
Jan	2.22	Jul	3.27
Feb	1.51	Aug	5.40
Mar	1.86	Sep	5.45
Apr	2.06	Oct	4.34
May	3.48	Nov	2.64
Jun	4.57	Dec	2.14

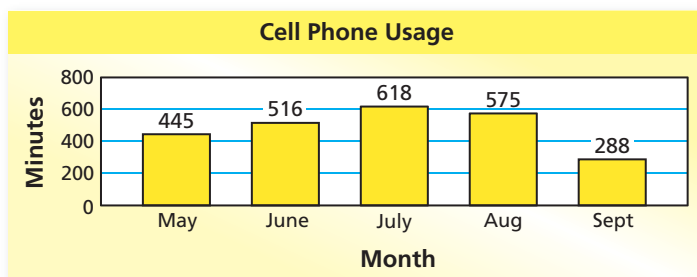
12. **RAINFALL** The table shows the monthly rainfall at a measuring station. What is the mean monthly rainfall?

13. **OPEN-ENDED** Create two different sets of data that have six values and a mean of 21.



14. **CELL PHONE** The bar graph shows your cell phone usage for five months.

- Which data value is an outlier? Explain.
- Find the mean with and without the outlier. Then describe how the outlier affects the mean.
- Describe a situation that could have caused the outlier in this problem.



15. **HEIGHT** The table shows the heights of the volleyball players from two schools. What is the difference between the mean heights of the two teams? Do outliers affect either mean? Explain.

	Player Height (inches)														
Dolphins	59	65	53	56	58	61	64	68	51	56	54	57			
Tigers	63	68	66	58	54	55	61	62	53	70	64	64	62	67	69

16. **REASONING** Make a dot plot of the data set 11, 13, 17, 15, 12, 18, and 12. Use the dot plot to explain how the mean is the point where the data set is balanced.
17. **ALLOWANCE** In your class, 7 students do not receive a weekly allowance, 5 students receive \$3, 7 students receive \$5, 3 students receive \$6, and 2 students receive \$8. What is the mean weekly allowance? Explain how you found your answer.
18. **Precision** A collection of 8 backpacks has a mean weight of 14 pounds. A different collection of 12 backpacks has a mean weight of 9 pounds. What is the mean weight of the 20 backpacks? Explain how you found your answer.



## Fair Game Review what you learned in previous grades & lessons

Evaluate the expression. (Section 1.3)

19.  $\frac{8 + 10}{2}$

20.  $\frac{26 + 34}{2}$

21.  $\frac{18 + 19}{2}$

22.  $\frac{14 + 17}{2}$

23. **MULTIPLE CHOICE** 60% of what number is 105? (Section 5.6)

(A) 63

(B) 175

(C) 630

(D) 1750

## 9.3 Measures of Center

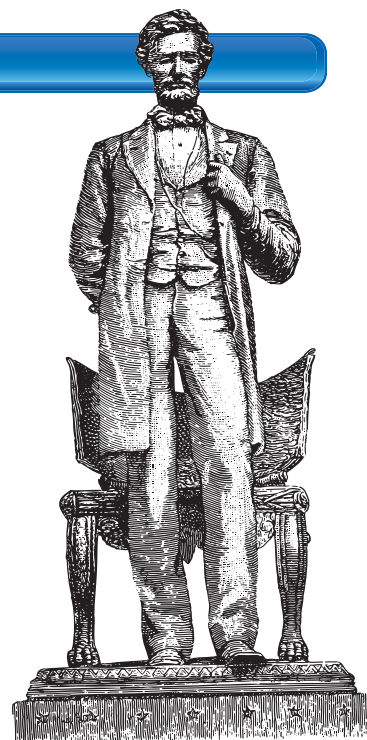
**Essential Question** In what other ways can you describe an average of a data set?

### 1 ACTIVITY: Finding a Median

Work with a partner.

- a. Write the total number of letters in the first and last names of 19 celebrities, historical figures, or people you know. Organize your data in a table. One person is already listed for you.

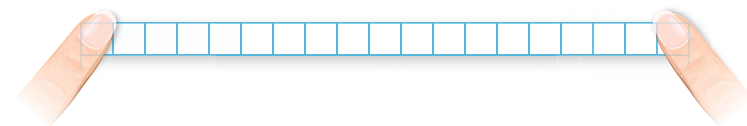
Person	Number of letters in first and last name
Abraham Lincoln	14



- b. Order the values in your data set from least to greatest. Then write the data on a strip of grid paper with 19 boxes.

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

- c. Place a finger on the square at each end of the strip. Move your fingers toward the center of the ordered data set until your fingers touch. On what value do your fingers touch?



- d. Now take your strip of grid paper and fold it in half. On what number is the crease? What do you notice? This value is called the *median*. How would you describe to another student what the median of a data set represents?
- e. How many values are greater than the median? How many are less than the median?
- f. Why do you think the median is considered an average of a data set?



#### Statistics

In this lesson, you will

- understand the concept of measures of center.
- find the median and mode of data sets.

Learning Standards

- 6.SP.2
- 6.SP.3
- 6.SP.5c

## 2

**a.** How many total letters are in your first name and last name? Add this value to the ordered data set in Activity 1. How many values are now in your data set?

- 

[illegible]

- 3

Math Practice 4

How can you use the dot plot to find the mode?

- Make a dot plot for the data set in Activity 2. Describe the distribution of the data.
- Which value occurs most often in the data set? This value is called the *mode*.
- Do you think a data set can have no mode or more than one mode? Explain.
- Do you think the mode always has to be a value in the data set? Explain.
- Why do you think the mode is considered an average of a data set?

4. **IN YOUR OWN WORDS** In what other ways can you describe an average of a data set?
5. Find the mean of your data set in Activity 2. Then compare the mean, median, and mode. Is there one measure that you think best represents your data set? Explain your reasoning.

Use what you learned about the median of a data set to complete Exercises 5 and 6 on page 407.



## 9.3 Lesson

### Key Vocabulary

measure of center,  
p. 404  
median, p. 404  
mode, p. 404

### Study Tip

The mode is the only measure of center that you can use to describe a set of data that is *not* made up of numbers.

A **measure of center** is a measure that describes the typical value of a data set. The mean is one type of measure of center. Here are two others.

## Key Ideas

### Median

**Words** Order the data. For a set with an odd number of values, the **median** is the middle value. For a set with an even number of values, the **median** is the mean of the two middle values.

**Numbers** Data: 5, 8, 9, 12, 14 The median is 9.

Data: 2, 3, 5, 7, 10, 11

The median is  $\frac{5+7}{2}$ , or 6.

### Mode

**Words** The **mode** of a data set is the value or values that occur most often. Data can have one mode, more than one mode, or no mode. When all values occur only once, there is no mode.

**Numbers** Data: 11, 13, 15, 15, 18, 21, 24, 24

The modes are 15 and 24.

## EXAMPLE 1 Finding the Median and Mode

### Bowling Scores

120	135	160	125	90
205	160	175	105	145

Find the median and mode of the bowling scores.

90, 105, 120, 125, 135, 145, 160, 160, 175, 205 Order the data.

**Median:**  $\frac{135 + 145}{2} = \frac{280}{2}$ , or 140 Add the two middle values and divide by 2.

**Mode:** 90, 105, 120, 125, 135, 145, 160, 160, 175, 205

The value 160 occurs most often.

∴ The median is 140. The mode is 160.

## On Your Own

Find the median and mode of the data.

- 20, 4, 17, 8, 12, 9, 5, 20, 13
- 100, 75, 90, 80, 110, 102

Now You're Ready  
Exercises 7–12

## EXAMPLE 2 Finding the Mode

### Favorite Types of Movies

Comedy Drama Horror  
 Horror Drama Horror  
 Comedy Comedy Action  
 Action Comedy Action  
 Horror Drama Comedy  
 Comedy Comedy Horror  
 Horror Comedy Action  
 Horror Action Drama

The list shows the favorite types of movies for students in a class. Organize the data in a frequency table. Then find the mode.

Type	Tally	Frequency
Action		5
Comedy	III	8
Drama		4
Horror	II	7

The number of tally marks is the frequency.

Make a tally for each vote.

Comedy received the most votes.

So, the mode is comedy.

### On Your Own

Now You're Ready  
 Exercises 14–15

3. One member of the class was absent and ends up voting for horror. Does this change the mode? Explain.

## EXAMPLE 3 Choosing the Best Measure of Center



Find the mean, median, and mode of the sneaker prices. Which measure best represents the data?

Mean:  $\frac{20 + 31 + 122 + 48 + 37 + 20 + 45 + 65}{8} = \frac{388}{8}$ , or 48.5

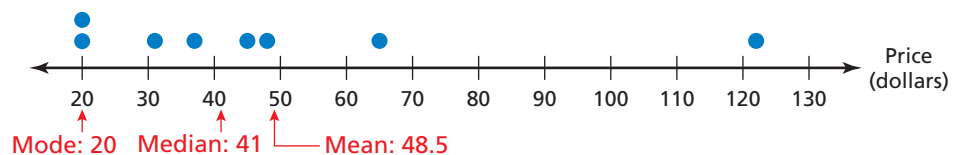
Median: 20, 20, 31, 37, 45, 48, 65, 122

Order from least to greatest.

$$\frac{37 + 45}{2} = \frac{82}{2}, \text{ or } 41$$

Mode: 20, 20, 31, 37, 45, 48, 65, 122

The value 20 occurs most often.



The median best represents the data. The mode is less than most of the data, and the mean is greater than most of the data.

### On Your Own

Now You're Ready  
 Exercises 17–20

Find the mean, median, and mode of the data. Choose the measure that best represents the data. Explain your reasoning.

4. 1, 93, 46, 48, 34, 194, 67, 55      5. 96, 150, 102, 87, 150, 75

## EXAMPLE 4 Removing an Outlier

Identify the outlier in Example 3. Find the mean, median, and mode without the outlier. Which measure does the outlier affect the most?

The price of \$122 is much greater than any other price. So, it is the outlier.

	Mean	Median	Mode
With Outlier (Example 3)	48.5	41	20
Without Outlier	38	37	20

∴ The mean is affected the most by the outlier.

### On Your Own

 **Now You're Ready**  
Exercises 21–22

6. The times (in minutes) it takes six students to travel to school are 8, 10, 10, 15, 20, and 45. Identify the outlier. Find the mean, median, and mode with and without the outlier. Which measure does the outlier affect the most?

## EXAMPLE 5 Changing the Values of a Data Set

The prices of six video games at an online store are shown in the table. The price of each game increases by \$4.98 when a shipping charge is included. How does this increase affect the mean, median, and mode?

Video Game Prices	
\$53.42	\$35.69
\$18.99	\$25.13
\$27.97	\$53.42

Make a new table by adding \$4.98 to each price. Then find the mean, median, and mode of both data sets.

Video Game Prices with Shipping Charge	
\$58.40	\$40.67
\$23.97	\$30.11
\$32.95	\$58.40

	Mean	Median	Mode
Original Price	35.77	31.83	53.42
Price with Shipping Charge	40.75	36.81	58.4

Compare:

**Mean:**  $40.75 - 35.77 = 4.98$

**Median:**  $36.81 - 31.83 = 4.98$

**Mode:**  $58.4 - 53.42 = 4.98$

∴ By increasing each video game price by \$4.98 for shipping, the mean, median, and mode all increase by \$4.98.

### On Your Own

7. **WHAT IF?** The store decreases the price of each video game by \$3. How does this decrease affect the mean, median, and mode?

## 9.3 Exercises



### Vocabulary and Concept Check

- NUMBER SENSE** Give an example of a data set that has no mode.
- WRITING** Which is affected most by an outlier: the mean, median, or mode? Explain.
- WHICH ONE DOESN'T BELONG** Which word does *not* belong with the other three? Explain.

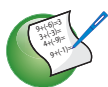
median

outlier

mode

mean

- NUMBER SENSE** A data set has a mean of 7, a median of 5, and a mode of 8. Which of the numbers 7, 5, and 8 *must* be in the data set? Explain.



### Practice and Problem Solving

Use grid paper to find the median of the data.

- 9, 7, 2, 4, 3, 5, 9, 6, 8, 0, 3, 8
- 16, 24, 13, 36, 22, 26, 22, 28, 25

Find the median and mode(s) of the data.

- 3, 5, 7, 9, 11, 3, 8
- 14, 19, 16, 13, 16, 14
- 93, 81, 94, 71, 89, 92, 94, 99
- 44, 13, 36, 52, 19, 27, 33
- 12, 33, 18, 28, 29, 12, 17, 4, 2
- 55, 44, 40, 55, 48, 44, 58, 67

- ERROR ANALYSIS** Describe and correct the error in finding the median of the data.



The median is 58.

63, 55, 49, 58, 50, 59, 51

Find the mode(s) of the data.

2 14.

Shirt Color		
Black	Blue	Red
Pink	Black	Black
Gray	Green	Blue
Blue	Blue	Red
Yellow	Blue	Blue
Black	Orange	Black
Black		

15.

Talent Show Acts		
Singing	Dancing	Comedy
Singing	Singing	Dancing
Juggling	Dancing	Singing
Singing	Poetry	Dancing
Comedy	Magic	Dancing
Poetry	Singing	Singing

- REASONING** In Exercises 14 and 15, can you find the mean and median of the data? Explain.



Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning.

3 17. 48, 12, 11, 45, 48, 48, 43, 32

18. 12, 13, 40, 95, 88, 7, 95

19. 2, 8, 10, 12, 56, 9, 5, 2, 4

20. 126, 62, 144, 81, 144, 103

Find the mean, median, and mode(s) of the data with and without the outlier. Describe the effect of the outlier on the measures of center.

4 21. 45, 52, 17, 63, 57, 42, 54, 58








22. 85, 77, 211, 88, 91, 84, 85

Find the mean, median, and mode(s) of the data.

23. 4.7, 8.51, 6.5, 7.42, 9.64, 7.2, 9.3

24.  $8\frac{1}{2}$ ,  $6\frac{5}{8}$ ,  $3\frac{1}{8}$ ,  $5\frac{3}{4}$ ,  $6\frac{5}{8}$ ,  $5\frac{1}{4}$ ,  $10\frac{5}{8}$ ,  $4\frac{1}{2}$

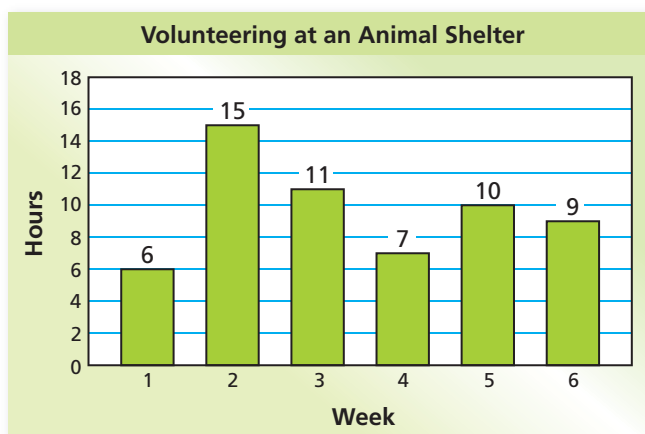
25. **WEATHER** The weather forecast for a week is shown.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
							
High	90° F	91° F	89° F	97° F	101° F	99° F	91° F
Low	74° F	78° F	77° F	77° F	83° F	78° F	72° F

- Find the mean, median, and mode(s) of the high temperatures. Which measure best represents the data? Explain your reasoning.
- Repeat part (a) for the low temperatures.

26. **RESEARCH** Find the unit costs of 10 different kinds of cereal. Choose one cereal whose unit cost will be an outlier.

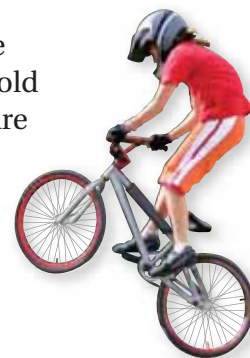
- Find the mean, median, and mode(s) of the data. Which measure best represents the data? Explain your reasoning.
- Identify the outlier in the data set. Find the mean, median, and mode(s) of the data set without the outlier. Which measure does the outlier affect the most?



27. **PROBLEM SOLVING** The bar graph shows the numbers of hours you volunteered at an animal shelter. What is the minimum number of hours you need to work in the seventh week to justify that you worked an average of 10 hours for the 7 weeks? Explain your answer using measures of center.

28. **REASONING** Why do you think the mode is the least frequently used measure to describe a data set? Explain.

29. **MOTOCROSS** The ages of the racers in a bicycle motocross race are 14, 22, 20, 25, 26, 17, 21, 30, 27, 25, 14, and 29. The 30-year-old drops out of the race and is replaced with a 15-year-old. How are the mean, median, and mode of the ages affected?



30. **CAMERAS** The data are the prices of several digital cameras at a store.

\$130 \$170 \$230 \$130  
\$250 \$275 \$130 \$185

- a. Does the price shown in the advertisement represent the prices well? Explain.
- b. Why might the store use this advertisement?
- c. In this situation, why might a person want to know the mean? the median? the mode? Explain.
31. **SALARIES** The table shows the monthly salaries for employees at a company.
- | Monthly Salaries (dollars) |      |      |      |      |
|----------------------------|------|------|------|------|
| 1940                       | 1660 | 1860 | 2100 | 1720 |
| 1540                       | 1760 | 1940 | 1820 | 1600 |
- a. Find the mean, median, and mode of the data.
- b. Each employee receives a 5% raise. Find the mean, median, and mode of the data with the raise. How does this increase affect the mean, median, and mode of the data?
- c. Use the original monthly salaries to calculate the annual salaries. Find the mean, median, and mode of the annual salaries. How are these values related to the mean, median, and mode of the monthly salaries?
32. **Critical Thinking** Consider the algebraic expressions  $3x$ ,  $9x$ ,  $4x$ ,  $23x$ ,  $6x$ , and  $3x$ . Assume  $x > 0$ .
- a. Find the mean, median, and mode.
- b. Is there an outlier? If so, what is it?



## Fair Game Review What you learned in previous grades & lessons

Find the value of the expression. (Section 1.1)

33.  $48 - 35$

34.  $188 - 123$

35.  $416 - 297$

36.  $6249 - 3374$

37. **MULTIPLE CHOICE** A shelf in your room can hold at most 30 pounds. There are 12 pounds of books already on it. Which inequality represents the number of pounds you can add to the shelf? (Section 7.6)

(A)  $x < 18$

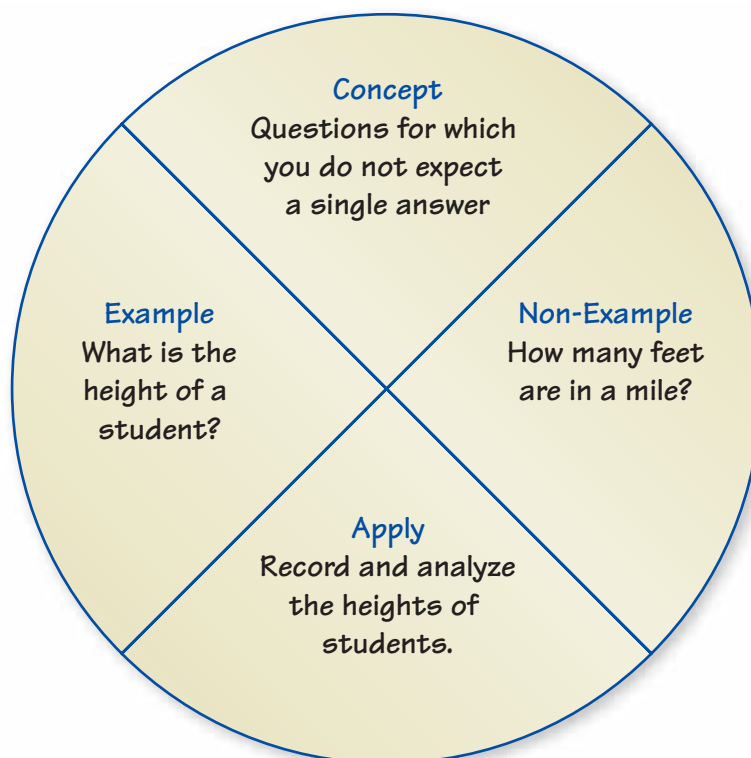
(B)  $x \geq 18$

(C)  $x \leq 42$

(D)  $x \leq 18$

You can use a **concept circle** to organize information about a concept. Here is an example of a concept circle for a statistical question.

## Statistical Question



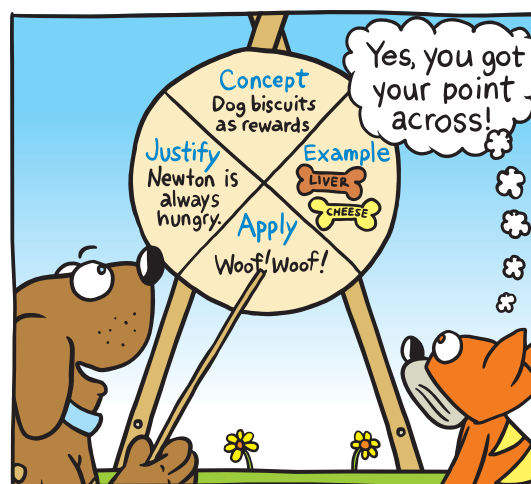
## On Your Own

Make concept circles to help you study these topics.

1. mean
2. outlier
3. measures of center
4. median
5. mode

After you complete this chapter, make concept circles for the following topics.

6. measures of variation
7. range
8. quartiles
9. interquartile range
10. mean absolute deviation



"Do you think this **concept circle** will help my owner understand that 'Speak' and 'Sit' need motivation?"

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data. (Section 9.1)

1.

Weight (grams)			
42	40	37	42
43	41	42	43
37	41	41	42

2.

Time (seconds)				
63	66	65	60	59
59	64	58	65	58
64	60	59	64	63

Find the mean of the data. (Section 9.2)

3.

Tour Dates	
May	III
June	HHH HHH
July	HHH HHH IIII
August	HHH HHH HHH III
September	HHH

4.

Scores	
Judge 1	8.9
Judge 2	9.4
Judge 3	8.6
Judge 4	9.1

Find the median and the mode(s) of the data. (Section 9.3)

5. 3, 5, 9, 11, 3

6. 24, 4, 37, 56, 6, 56, 45

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning. (Section 9.3)

7. 47, 147, 24, 47, 38, 42

8. 34, 57, 58, 56, 21

Hours of Exercise				
5	1	5	3	5
4	5	2	5	4
3	4	6	5	6

9. **EXERCISE** You conduct a survey to answer: “How many hours does a sixth-grade student spend exercising during a week?” The table shows the results. (Section 9.1)

a. Is this a statistical question? Explain.

b. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

c. Use the distribution of the data to answer the question.

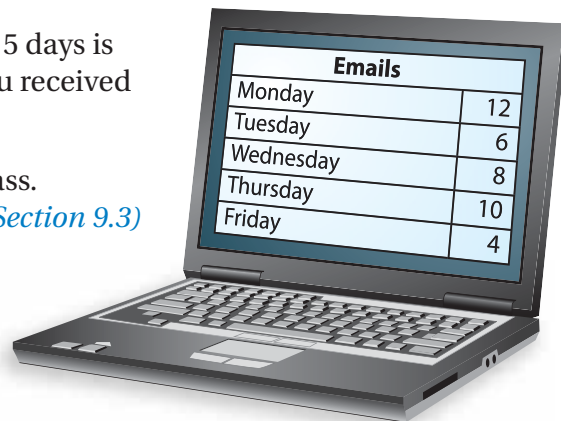
10. **EMAILS** The number of emails you received in 5 days is shown. What is the mean number of emails you received per day? (Section 9.2)

11. **QUIZZES** The data are your quiz scores for a class. Find the median and the mode of the data. (Section 9.3)

18, 19, 17, 14, 20, 20, 15, 21

12. **MUSIC** The data are the lengths of the songs (in minutes) on your new CD. Which measure of center best represents the data with and without the outlier? Explain. (Section 9.3)

2.2, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 14.2



Emails	
Monday	12
Tuesday	6
Wednesday	8
Thursday	10
Friday	4



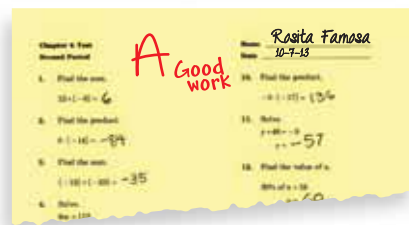
## 9.4 Measures of Variation

**Essential Question** How can you describe the spread of a data set?

### 1 ACTIVITY: Interpreting Statements

Work with a partner. There are 24 students in your class. Your teacher makes the following statements:

- “The exam scores range from 75% to 96%.”
- “Most of the students received high scores.”



- What do you think the first statement means? Explain.
- In the first statement, is your teacher describing the center of the data set? If not, what do you think your teacher is describing?
- What do you think the scores are for most of the students in the class? Explain your reasoning.
- Use your teacher's statements to make a dot plot that can represent the distribution of the exam scores of the class.

### 2 ACTIVITY: Grouping Data

Work with a partner. The numbers of U.S. states visited by each student in a sixth grade class are shown.

- Between what values do the data range?
- Write the ordered data values on a strip of grid paper and fold it to find the median. How many values are greater than the median? How many are less than the median?

Number of States Visited			
1	7	5	2
11	6	3	20
4	18	1	6
2	7	1	8
10	2	12	5
	3	21	

- REPEATED REASONING** Fold the strip in half again. On what values are the two new creases? What do you think these values represent?
- Into how many parts did you divide the data set? How many data values are in each part?
- Graph the median and the values you found in parts (a) and (c) on a number line. Are the distances the same between these points?
- How can you use these values to describe the spread of the data?



#### Statistics

In this lesson, you will

- find the range of data sets.
- find the interquartile range of data sets.
- check for outliers in data sets.

Learning Standards

- 6.SP.2
- 6.SP.3
- 6.SP.5c

### 3 ACTIVITY: Adding a Value to a Data Set

Work with a partner. A new student joins the class in Activity 2. The new student has visited 41 states.

- Add this value to the ordered data set in Activity 2. Does your answer to part (a) change? Explain.
- How does the distribution of the data change when this value is added? Explain your reasoning.
- How does adding this value affect the values on your number line in part (e) of Activity 2?

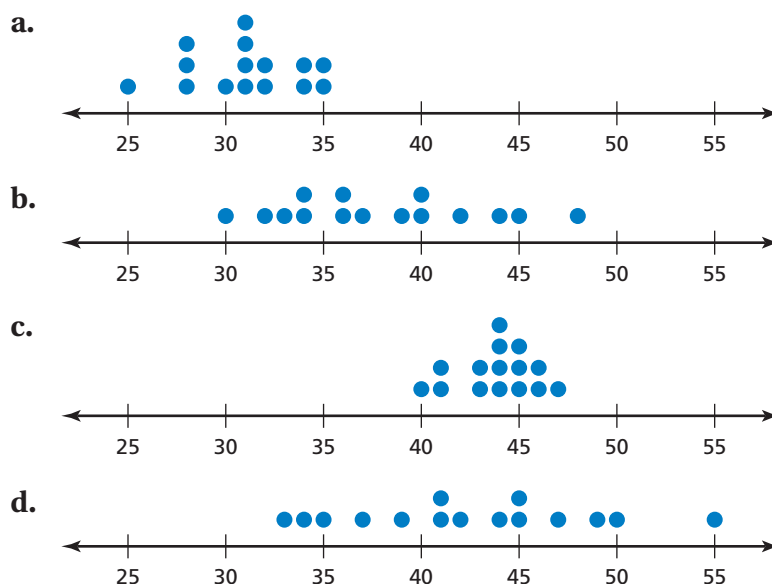
### 4 ACTIVITY: Analyzing Data Sets

Work with a partner. Identify the data set that is the least spread out and the data set that is the most spread out. Explain your reasoning.

#### Math Practice 1

##### Analyze Givens

How can you use the given information to determine how spread out the data are?



### What Is Your Answer?

- IN YOUR OWN WORDS** How can you describe the spread of a data set?
- Make a dot plot of the data set in Activity 2. Describe any similarities between the dot plot and the number line in part (e).

#### Practice

Use what you learned about variation to complete Exercises 4 and 5 on page 416.

A **measure of variation** is a measure that describes the distribution of a data set. A simple measure of variation to find is the *range*. The **range** of a data set is the difference between the greatest value and the least value.

## EXAMPLE 1 Finding the Range

### Key Vocabulary

measure of variation,  
p. 414  
range, p. 414  
quartiles, p. 414  
first quartile, p. 414  
third quartile, p. 414  
interquartile range,  
p. 414

The table shows the lengths of several Burmese pythons captured for a study. Find and interpret the range of their lengths.

To find the least and the greatest values, order the lengths from least to greatest.

5, 6.25, 8, 10, 11, 12.5, 14, 15.5, 16.25, 18.5

The least value is 5. The greatest value is 18.5.

So, the range of the lengths is  $18.5 - 5$ , or 13.5 feet. This means that the lengths vary by no more than 13.5 feet.

Lengths (feet)	
18.5	8
11	10
14	15.5
12.5	6.25
16.25	5

Now You're Ready  
Exercises 6–9

## On Your Own

- The ages of people in line for a roller coaster are 15, 17, 21, 32, 41, 30, 25, 52, 16, 39, 11, and 24. Find and interpret the range of their ages.

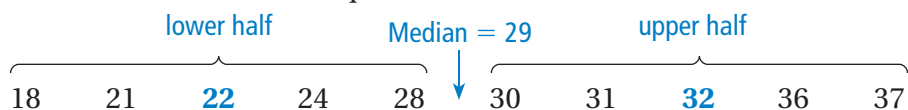
## Key Ideas

### Reading

The first quartile can also be called the *lower quartile*. The third quartile can also be called the *upper quartile*.

### Quartiles

The **quartiles** of a data set divide the data into four equal parts. Recall that the median (second quartile) divides the data set into two halves.

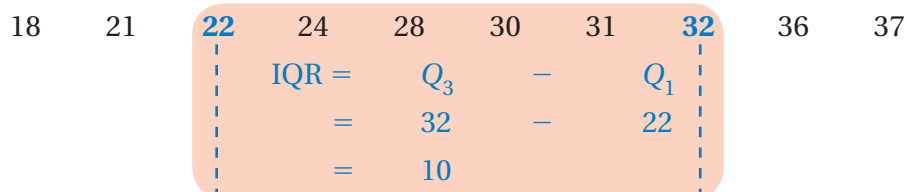


The median of the lower half is the **first quartile,  $Q_1$** .

The median of the upper half is the **third quartile,  $Q_3$** .

### Interquartile Range (IQR)

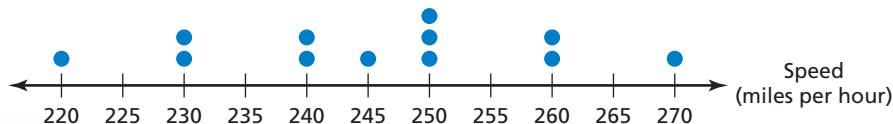
The difference between the third quartile and the first quartile is called the **interquartile range**. The IQR represents the range of the middle half of the data and is another measure of variation.



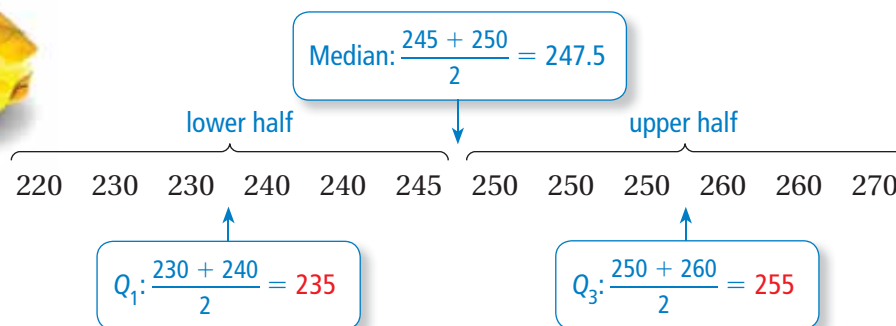
## EXAMPLE 2

## Finding the Interquartile Range

The dot plot shows the top speeds of 12 sports cars. Find and interpret the interquartile range of the data.

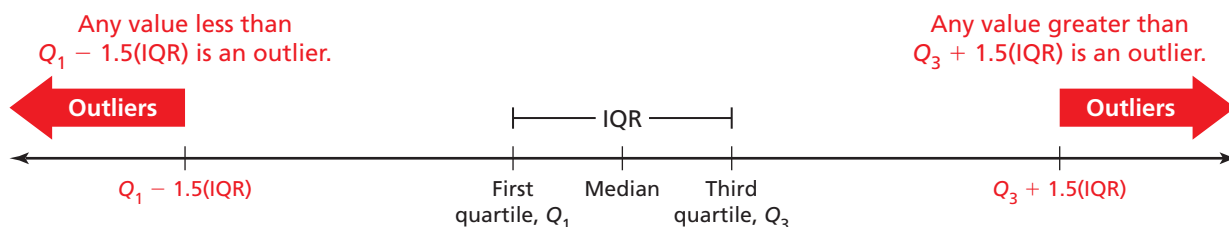


Order the speeds from slowest to fastest. Find the quartiles.



So, the interquartile range is  $255 - 235 = 20$ . This means that the middle half of the speeds vary by no more than 20 miles per hour.

You can use the quartiles and the interquartile range to check for outliers.



## EXAMPLE 3

## Checking for Outliers

Check for outliers in the data set in Example 2.

$Q_1 - 1.5(IQR)$	Outlier boundaries	$Q_3 + 1.5(IQR)$
$235 - 1.5(20)$	Substitute values.	$255 + 1.5(20)$
205	Simplify.	285

There are no speeds less than 205 miles per hour or greater than 285 miles per hour. So, the data set has no outliers.

## On Your Own

**Now You're Ready**  
Exercises 11–14  
and 17

2. The number of pages in each of an author's novels is shown.

356, 364, 390, 468, 400, 382, 376, 396, 350

- Find and interpret the interquartile range of the data.
- Does this data set contain any outliers? Justify your answer.





## Vocabulary and Concept Check

- VOCABULARY** How are measures of center different from measures of variation?
- VOCABULARY** How many quartiles does a data set have?
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

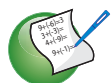
53, 47, 60, 45, 62, 59, 65, 50, 56, 48

What is the interquartile range of the data?

What is the range of the data?

What is the range of the middle half of the data?

What is the difference between the third quartile and the first quartile?



## Practice and Problem Solving

Use grid paper to find the median of the data. Then find the median of the lower half and the median of the upper half of the data. Describe the spread of the data.

- 5, 8, 10, 1, 7, 6, 15, 8, 6
- 82, 62, 95, 81, 89, 51, 72, 56, 97, 98, 79, 85

Find the range of the data.

- 26, 21, 27, 33, 24, 29
- 52, 40, 49, 48, 62, 54, 44, 58, 39
- 133, 117, 152, 127, 168, 146, 174
- 4.8, 5.5, 4.2, 8.9, 3.4, 7.5, 1.6, 3.8

- ERROR ANALYSIS** Describe and correct the error in finding the range of the data.



49, 48, 51, 41, 35, 44, 38

The range is  $49 - 38$ , or 11.

Find the median, first quartile, third quartile, and interquartile range of the data.

- 40, 33, 37, 54, 41, 34, 27, 39, 35
- 84, 75, 90, 87, 99, 91, 85, 88, 76, 92, 94
- 132, 127, 106, 140, 158, 135, 129, 138
- 38, 55, 61, 56, 46, 67, 59, 75, 65, 58

- PAPER AIRPLANE** The table shows the distances traveled by a paper airplane. Find and interpret the range and the interquartile range of the distances.

Distances (feet)			
$13\frac{1}{2}$	$21\frac{1}{2}$	21	$16\frac{3}{4}$
$10\frac{1}{4}$	19	32	$26\frac{1}{2}$
29	$16\frac{1}{4}$	$28\frac{1}{2}$	$18\frac{1}{2}$



16. **WRITING** Consider a data set that has no mode. Which measure of variation is greater, the range or the interquartile range? Explain your reasoning.
- 3 17. **OUTLIERS** Use the interquartile range to identify any outliers in Exercises 11–14.
18. **REASONING** How does an outlier affect the range of a data set? Explain.
19. **BASKETBALL** The table shows the numbers of points scored by players on a basketball team.

Points Scored					
21	53	74	82	84	93
103	108	116	122	193	

- a. Find the range and the interquartile range of the data.
- b. Use the interquartile range to identify the outlier(s) in the data set. Find the range and the interquartile range of the data set without the outlier(s). Which measure did the outlier(s) affect more?
20. **STRUCTURE** Two data sets have the same range. Can you assume that the interquartile ranges of the two data sets are about the same? Give an example to justify your answer.



21. **SINGING** The tables show the ages of the finalists for two reality singing competitions.

- a. Find the mean, median, range, and interquartile range of the ages for each show. Compare the results.
- b. A 21-year-old is voted off Show A, and the 36-year-old is voted off Show B. How do these changes affect the measures in part (a)? Explain.

Ages for Show A		Ages for Show B	
18	17	21	20
15	21	23	13
22	16	15	18
18	28	17	22
24	21	36	25

22. **Open-Ended** Create a set of data with 7 values that has a mean of 30, a median of 26, a range of 50, and an interquartile range of 36.



## Fair Game Review What you learned in previous grades & lessons

Find the mean of the data. (Section 9.2)

23. 8, 14, 22, 7, 2, 11, 25, 7, 5, 9

24. 55, 64, 58, 43, 49, 67

25. **MULTIPLE CHOICE** What is the surface area of the rectangular prism? (Section 8.2)

- (A)  $62 \text{ m}^2$  (B)  $72 \text{ m}^2$   
(C)  $88 \text{ m}^2$  (D)  $124 \text{ m}^2$



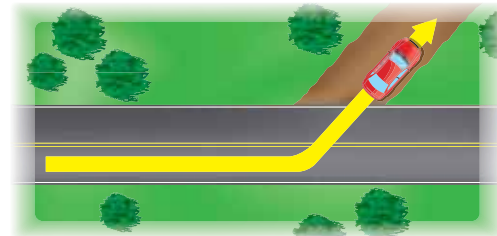
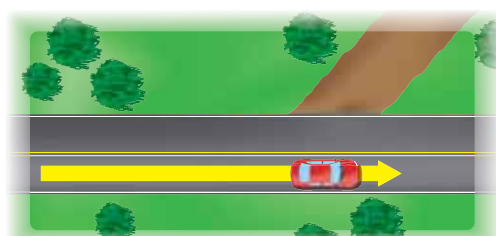
## 9.5 Mean Absolute Deviation

**Essential Question** How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?

### The Meaning of a Word ● Deviate

When you **deviate** from something,

you stray or depart from the normal course of action.



### 1 ACTIVITY: Finding Distances From the Mean

Work with a partner. The table shows the exam scores of 14 students in your class.

Exam Scores			
Ben	89	Mike	95
Emma	86	Hong	96
Jeremy	80	Rob	92
Pete	80	Amy	90
Ryan	96	Sue	76
Dan	94	Kim	84
Lucy	89	Heather	85



#### Statistics

In this lesson, you will

- understand the meaning of *mean absolute deviation*.
- find the mean absolute deviation of data sets.

Learning Standards

6.SP.2  
6.SP.3  
6.SP.5c

- What is the mean exam score?
- Make a dot plot of the data. Place an “X” on the number line to represent the mean.
- Is the number of exam scores that are greater than the mean equal to the number of exam scores that are less than the mean? Explain.
- Which exam score *deviates* the most from the mean? Which exam score *deviates* the least from the mean? Explain how you found your answers.
- Overall, do you think the exam scores are *close* to the mean or *far away* from the mean? Explain your reasoning.

## 2 ACTIVITY: Using Distances from the Mean

Work with a partner. Use the information in Activity 1.

- a. Complete the table below. Add rows if needed. Be sure to find the sum of the values in the last column of the table.

### Math Practice 2

#### Use Operations

What operation can you use to find the distance from the mean? Explain.

Student with Score Less Than the Mean	Exam Score	Distance from the Mean
Sum:		

- b. Create a table similar to the one above for students with scores *greater than* the mean.
- c. **LOGIC** What do you notice about the sums you found in your tables? Why do you think this happens?

## 3 ACTIVITY: Interpreting Distances from the Mean

Work with a partner.

- a. **LOGIC** Add the sums you found in your tables in Activity 2. Divide that amount by the total number of students. Round your result to the nearest tenth.

In your own words, what do you think this value represents?

- b. **REASONING** In a data set, what do you think it means when the value you found in part (a) is close to 0? Explain.

### What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?
5. **REASONING** Find the range and the interquartile range of the data set in Activity 1. What do you think it means when these values are close to 0? Explain.

### Practice

Use what you learned about distances from the mean to complete Exercises 3 and 4 on page 422.

**Key Vocabulary**

mean absolute deviation, p. 420

Another measure of variation is the *mean absolute deviation*. The **mean absolute deviation** is an average of how much data values differ from the mean.

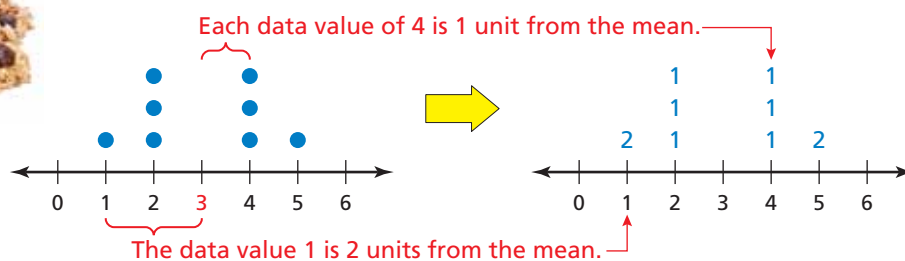
**Key Idea****Finding the Mean Absolute Deviation (MAD)****Step 1:** Find the mean of the data.**Step 2:** Find the distance between each data value and the mean.**Step 3:** Find the sum of the distances in Step 2.**Step 4:** Divide the sum in Step 3 by the total number of data values.**EXAMPLE 1 Finding the Mean Absolute Deviation**

You record the numbers of raisins in 8 scoops of cereal. Find and interpret the mean absolute deviation of the data.

1, 2, 2, 2, 4, 4, 4, 5

**Step 1:** Mean =  $\frac{1 + 2 + 2 + 2 + 4 + 4 + 4 + 5}{8} = \frac{24}{8} = 3$

**Step 2:** You can use a dot plot to organize the data. Replace each dot with its distance from the mean.



**Step 3:** The sum of the distances is  $2 + 1 + 1 + 1 + 1 + 1 + 1 + 2 = 10$ .

**Step 4:** The mean absolute deviation is  $\frac{10}{8} = 1.25$ .

So, the data values differ from the mean by an average of 1.25 raisins.

**On Your Own**

Now You're Ready  
Exercises 5–8

1. Find and interpret the mean absolute deviation of the data.

5, 8, 8, 10, 13, 14, 16, 22



## EXAMPLE 2 Real-Life Application



Mendoza		
Date	Win/Loss	Runs
Aug 8	-	4
Aug 3	-	6
Jul 29	L	6
Jul 24	W	0
Jul 13	L	8
Jul 8	-	4
Jul 7	L	5
Jul 2	-	0
Jun 27	W	2
Jun 22	W	0

The smartphones show the numbers of runs allowed by two pitchers in their last 10 starts.

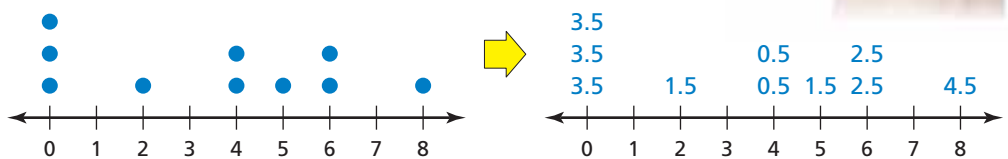
- a. Find the mean, median, and mean absolute deviation of the numbers of runs allowed for each pitcher.

Order the runs allowed for Mendoza:  
0, 0, 0, 2, 4, 4, 5, 6, 6, 8.

$$\text{Mean} = \frac{35}{10} = 3.5$$

$$\text{Median} = \frac{4 + 4}{2} = 4$$

Mean absolute deviation:



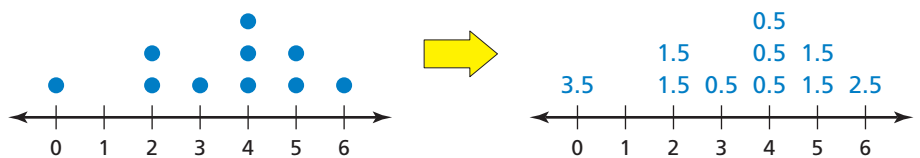
$$\text{The mean absolute deviation is } \frac{24}{10} = 2.4$$

Order the runs allowed for Rodriguez: 0, 2, 2, 3, 4, 4, 4, 5, 5, 6.

$$\text{Mean} = \frac{35}{10} = 3.5$$

$$\text{Median} = \frac{4 + 4}{2} = 4$$

Mean absolute deviation:



$$\text{The mean absolute deviation is } \frac{14}{10} = 1.4.$$

### Study Tip

The greater the mean absolute deviation, the greater the variation of the data.

- b. Which measure can you use to distinguish the data? What can you conclude about the pitchers from this measure?

You cannot use the measures of center to distinguish the data because they are the same for each data set. The measure of variation, MAD, is 2.4 for Mendoza and 1.4 for Rodriguez. This indicates that the data for Rodriguez has less variation.

- Using the MAD to distinguish the data, you can conclude that Rodriguez is more consistent than Mendoza.

### On Your Own

2. **WHAT IF?** Mendoza allows 4 runs in the next game. How would you expect the mean absolute deviation to change? Explain.



## Vocabulary and Concept Check

- REASONING** Describe a data set that has a mean absolute deviation of 0.
- WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

range

interquartile range

mean

mean absolute deviation



## Practice and Problem Solving

Find the average distance each data value in the set is from the mean. Round your answer to the nearest tenth, if necessary.

- Model years of used cars on a lot: 2010, 2002, 2005, 2007, 2001
- Prices of kites at a shop: \$7, \$20, \$9, \$35, \$12, \$15, \$7, \$10, \$20, \$25

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth, if necessary.

1

5. **Prices of Microphones (dollars)**

25	28	20	22	32
28	35	34	30	36

6. **Heights of 10-Year-Old Octuplets (inches)**

61	61	61	61
61	61	61	61

7. **Capacities of Stadiums (thousands of people)**

101.5	95.4	109.8
98.7	92.3	104.7

8. **Numbers of Visitors to a Website During a Week**

103	115	124	125
171	165	170	

9. **ERROR ANALYSIS** Describe and correct the error in finding the mean absolute deviation of the data set 35, 40, 38, 32, 42, and 41.



$$\text{mean} = \frac{35 + 40 + 38 + 32 + 42 + 41}{6} = 38$$

$$\text{MAD} = \frac{3 + 2 + 6 + 4 + 3}{5} = 3.6$$

So, the values differ from the mean by an average of 3.6.

10. **MUSEUMS** The data set shows the admission prices at several museums.

\$20, \$20, \$16, \$12, \$15, \$25, \$11

Find and interpret the range, interquartile range, and mean absolute deviation of the data.



11. **MENU** The table shows the prices of the five most-expensive and least-expensive dishes on a menu. Find the MAD of each data set. Then compare their variations.

Five Most-Expensive Dishes					Five Least-Expensive Dishes				
\$28	\$30	\$28	\$39	\$25	\$7	\$7	\$10	\$8	\$12

12. **COINS** The data sets show the years of the coins in two collections.

Derek's collection: 1950, 1952, 1908, 1902, 1955, 1954, 1901, 1910

Paul's collection: 1929, 1935, 1928, 1930, 1925, 1932, 1933, 1920

Find the measures of center and the measures of variation for each data set. Compare the measures. What can you conclude?



13. **PROBLEM SOLVING** You survey students in your class about the number of movies they watched last month. The results are shown in the table.

Movies Watched			
7	5	14	5
6	9	10	12
15	4	5	8
11	10	9	2

- a. Find the measures of center and the measures of variation for the data.
- b. A new student joins the class who watched 21 movies last month. Is 21 an outlier? How does including this value affect the measures of center and the measures of variation? Explain.

**REASONING** Which data set do you think would have the greater mean absolute deviation? Explain your reasoning.

14. guesses for number of gumballs in a jar  
guesses for number of baseballs in a jar
15. monthly rainfall amounts in a city  
monthly amounts of water used in a home
16. **REASONING** The MAD of a data set is considered a more reliable measure of variation than the range or the interquartile range. Why do you think this is true?
17. **Critical Thinking** Add and subtract the MAD from the mean in the original data set in Exercise 13.
- a. What percent of the values are within one MAD of the mean? two MADs of the mean? Which values are more than twice the MAD from the mean?
- b. What do you notice as you get more and more MADs away from the mean? Explain.



## Fair Game Review What you learned in previous grades & lessons

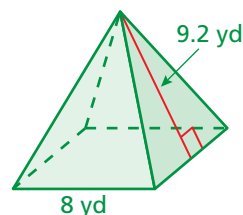
Find the mean, median, and mode(s) of the data. (Section 9.2 and Section 9.3)

18. 4, 6, 7, 9, 6, 4, 5, 6, 8, 10

19. 1.2, 1.7, 1.7, 2.1, 1.4, 1.2, 1.9

20. **MULTIPLE CHOICE** What is the surface area of the square pyramid? (Section 8.3)

- (A)  $100.8 \text{ yd}^2$  (B)  $147.2 \text{ yd}^2$
- (C)  $211.2 \text{ yd}^2$  (D)  $368 \text{ yd}^2$



# 9.4–9.5 Quiz



Find the range of the data. (Section 9.4)

1. 35, 76, 43, 58, 34, 67
2. 19, 21, 22, 22, 19, 25, 24, 23, 24

Find the median, first quartile, third quartile, and interquartile range of the data. (Section 9.4)

3. 56, 48, 72, 37, 35, 42, 48, 33, 28
4. 95, 14, 86, 62, 55, 46, 28, 37, 33, 70, 31

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary. (Section 9.5)

5. **Ages of Television Show Viewers (years)**

29	18	26	33
33	22	34	26

6. **Prices of Houses (thousands of dollars)**

80	120	95	240	140
75	135	110	90	125



7. **AMUSEMENT PARKS** The data set shows the admission prices at several amusement parks.

\$65, \$70, \$40, \$55, \$35, \$40, \$60

Find and interpret the range, interquartile range, and mean absolute deviation of the data. (Section 9.4 and Section 9.5)

8. **TEACHING EXPERIENCE** The tables show the years of teaching experience of faculty members at two schools. (Section 9.4)

- a. Find the mean, median, range, and interquartile range of the years of experience for each school. Compare the results.
- b. The teacher with 11 years of experience leaves School A, and the teacher with 33 years of experience retires from School B. How does this affect the measures in part (a)? Explain.

**School A:  
Teaching  
Experience  
(years)**

5	11
10	22
7	8
8	6
10	35

**School B:  
Teaching  
Experience  
(years)**

4	15
6	12
10	33
12	20
8	7

9. **BOOK CLUB** You survey the students in your book club about the number of books they read last summer. The results are shown in the table. (Section 9.4 and Section 9.5)

**Books Read**

8	14	15	9
6	12	9	13
11	11	7	5
12	6	10	8

- a. Find the measures of center and the measures of variation for the data.
- b. A new student who read 18 books last summer joins the club. Is 18 an outlier? How does adding this value to the data set affect the measures of center and variation? Explain.

# 9 Chapter Review



## Review Key Vocabulary

statistics, p. 392

statistical question, p. 392

mean, p. 398

outlier, p. 399

measure of center, p. 404

median, p. 404

mode, p. 404

measure of variation, p. 414

range, p. 414

quartiles, p. 414

first quartile, p. 414

third quartile, p. 414

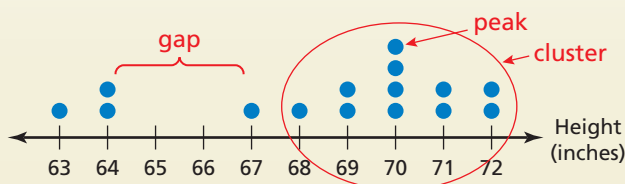
interquartile range, p. 414

mean absolute deviation,  
p. 420

## Review Examples and Exercises

### 9.1 Introduction to Statistics (pp. 390–395)

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.



Heights (inches)				
70	71	70	72	69
68	69	71	64	70
64	63	72	70	67

### Exercises

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

1.

Distance (feet)			
56	55	56	57
58	54	51	55
51	56	49	56

2.

Weight (pounds)				
83	88	89	90	89
91	89	84	90	92
90	88	89	83	88

### 9.2 Mean (pp. 396–401)

Find the mean of 5, 9, 10, 6, 6, and 12.

$$\begin{aligned} \text{mean} &= \frac{5 + 9 + 10 + 6 + 6 + 12}{6} \\ &= \frac{48}{6}, \text{ or } 8 \end{aligned}$$

sum of the data  
number of values  
Simplify.

### Exercises

Find the mean of the data.

3. 4, 5, 7, 14, 17, 12, 18

4. 15, 5, 8, 12, 5, 9, 4, 10, 2, 11



### 9.3 Measures of Center (pp. 402–409)

Find the median and the mode of the movie lengths in the table.

Order the data from least to greatest.

Movie Lengths (minutes)		
91	112	126
142	122	112
92	144	

**Median:**

91, 92, 112, **112**, **122**, 126, 142, 144

$$\frac{112 + 122}{2} = \frac{234}{2}, \text{ or } 117$$

**Mode:**

91, 92, **112**, **112**, 122, 126, 142, 144

The value **112** occurs most often.

∴ The median is 117 minutes, and the mode is 112 minutes.

### Exercises

Find the median and the mode(s) of the data.

5. 8, 8, 6, 8, 4, 5, 6

6. 24, 74, 61, 29, 38, 27, 68, 54

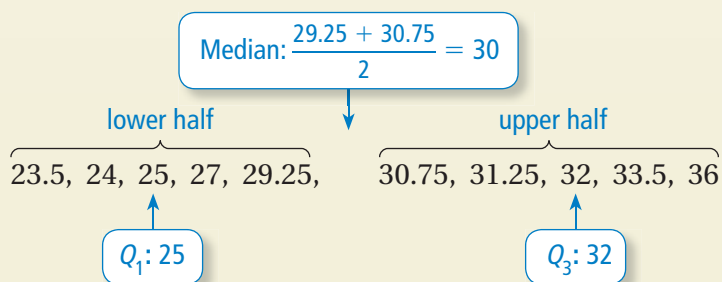
### 9.4 Measures of Variation (pp. 412–417)

The table shows the weights of several adult emperor penguins. (a) Find and interpret the range. (b) Find and interpret the interquartile range. (c) Check for outliers.

a. Ordered from least to greatest, the weights are 23.5, 24, 25, 27, 29.25, 30.75, 31.25, 32, 33.5, and 36.

∴ So, the range of the weights is  $36 - 23.5$ , or 12.5 kilograms. The weights vary by no more than 12.5 kilograms.

b. Find the quartiles.



∴ So, the interquartile range is  $32 - 25 = 7$ . This means that the middle half of the weights vary by no more than 7 kilograms.

c. Calculate the outlier boundaries.

$$Q_1 - 1.5(\text{IQR}) = 25 - 1.5(7) = 14.5$$

$$Q_3 + 1.5(\text{IQR}) = 32 + 1.5(7) = 42.5$$

∴ There are no weights less than 14.5 kilograms or greater than 42.5 kilograms. So, the data set has no outliers.

Weights (kilograms)

25	27
36	23.5
33.5	31.25
30.75	32
24	29.25



## Exercises

Find the range of the data.

7. 45, 76, 98, 21, 52, 39

8. 95, 63, 52, 8, 93, 16, 42, 37, 62

Find the median, first quartile, third quartile, and interquartile range of the data.

9. 28, 46, 25, 76, 18, 25, 47, 83, 44

10. 14, 25, 97, 55, 66, 28, 92, 38, 94

9.5

## Mean Absolute Deviation (pp. 418–423)

You record the prices of 8 printers. Find and interpret the mean absolute deviation of the data.

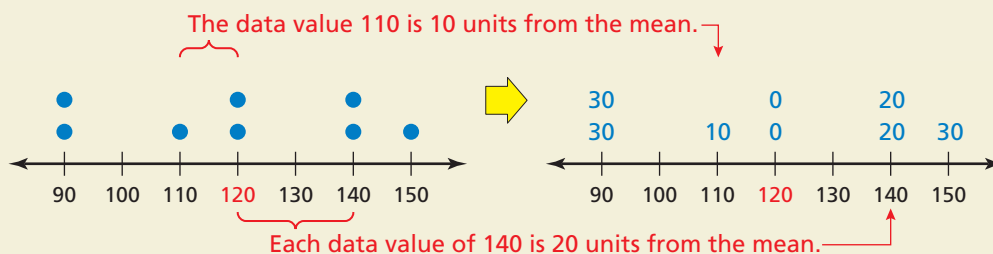
\$120, \$150, \$90, \$110,

\$140, \$120, \$140, \$90



**Step 1:** Mean =  $\frac{120 + 150 + 90 + 110 + 140 + 120 + 140 + 90}{8} = \frac{960}{8} = 120$

**Step 2:** Use a dot plot to organize the data. Replace each dot with its distance from the mean.



**Step 3:** The sum of the distances is  $30 + 30 + 10 + 0 + 0 + 20 + 20 + 30 = 140$ .

**Step 4:** The mean absolute deviation is  $\frac{140}{8} = 17.50$ .

∴ The data values differ from the mean by an average of \$17.50.

## Exercises

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary.

11.

Shoe Sizes

6	8.5	6	9
10	7	8	9.5

12.

Prices of Monitors (dollars)

130	150	190	100	175
120	165	140	180	190

# 9 Chapter Test



Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

1.

Time (minutes)			
33	40	32	40
39	38	40	39
38	39	39	33

2.

Temperature (°F)				
81	81	80	82	81
83	76	83	76	80
75	83	82	82	81

Find the mean, median, and mode(s) of the data.

3. 2, 7, 7, 12, 4

4. 4, 5, 7, 5, 9, 9, 10, 6

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning.

5. 5, 6, 4, 24, 18

6. 46, 27, 94, 56, 53, 65, 43

Find the range of the data.

7. 24, 56, 9, 83, 77, 14

8. 43, 12, 55, 91, 25, 86, 84, 23, 1

Find the median, first quartile, third quartile, and interquartile range of the data.

9. 32, 58, 19, 36, 44, 57, 11, 26, 74

10. 36, 24, 49, 32, 37, 28, 38, 40, 39

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary.

11.

Distances Driven (miles)			
312	286	196	201
158	225	206	192

12.

Prices of Sunglasses (dollars)				
15	8	19	20	18
20	22	14	10	15

13. **HOTEL** The table shows the numbers of guests at a hotel on different days.

Numbers of Guests					
66	58	90	57	63	55
60	62	56	54	72	

a. Find the range and the interquartile range of the data.

b. Use the interquartile range to identify the outlier(s) in the data set. Find the range and the interquartile range of the data set without the outlier(s). Which measure did the outlier(s) affect more?

14. **JOBS** The data sets show the numbers of hours worked each week by two friends for several weeks.

Greg's hours: 9, 18, 12, 6, 9, 21, 3, 12

Tom's hours: 12, 18, 15, 16, 14, 12, 15, 18

Find the measures of center and the measures of variation for each data set. Compare the measures. What can you conclude?

# 9 Standards Assessment

1. What is the value of the expression below?  
(6.NS.1)

$$8\frac{4}{9} \div 4\frac{2}{3}$$

- A.  $1\frac{17}{21}$  C.  $32\frac{8}{27}$   
B.  $2\frac{2}{3}$  D.  $39\frac{11}{27}$
2. What is the value of the expression below?  
(6.NS.3)

$$4.18 + 6.225 + 5.7$$

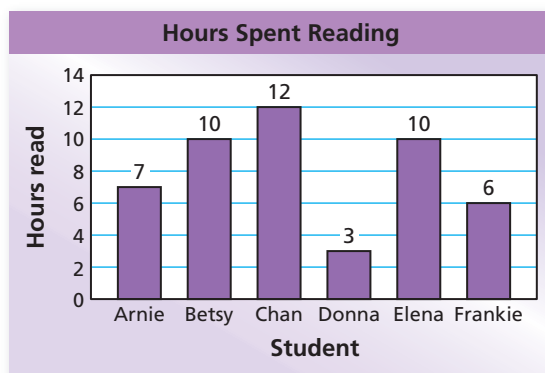
- F. 15.005 H. 16.005  
G. 15.105 I. 16.105
3. One number is missing from the data set in the box below.



18, 24, 22, 30, 26, \_\_\_\_, 25

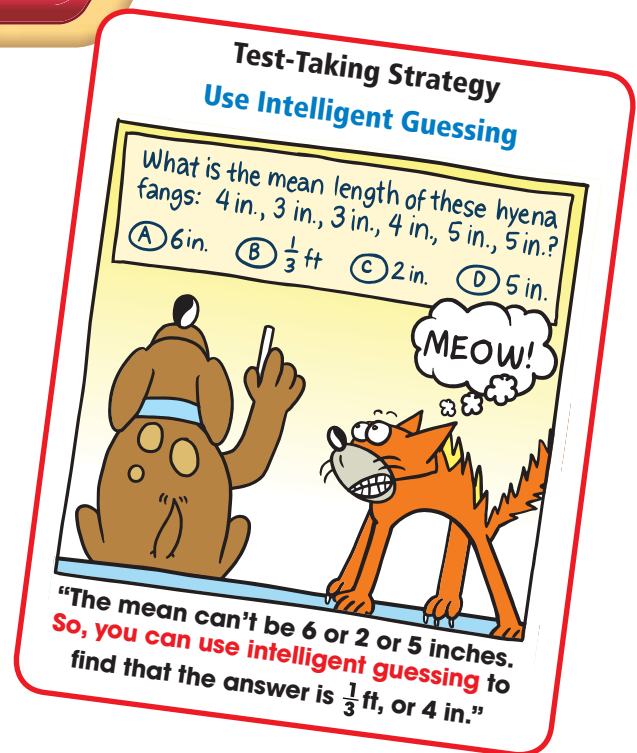
The median of the data set is 24. What is the greatest possible value of the missing number? (6.SP.5c)

4. The number of hours that each of 6 students spent reading last week is shown in the bar graph below.



For the data in the bar graph, which measure is the *least*? (6.SP.5c)

- A. mean C. mode  
B. median D. range



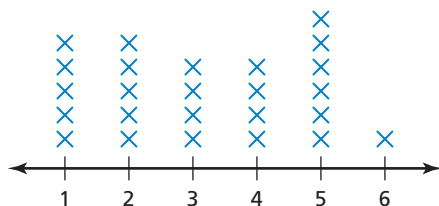
5. You go to a beach and collect buckets of shells. Of the many shells you have collected, you notice the following.

- 9% of the seashells are auger shells.
- $\frac{1}{8}$  of the seashells are coquina shells.
- 11% of the seashells are rough scallop shells.
- 0.1 of the seashells are fighting conch shells.

Which list correctly shows the types of shells in order from least to greatest? (6.NS.6c)

- F. auger, coquina, rough scallop, fighting conch
- G. fighting conch, coquina, auger, rough scallop
- H. fighting conch, auger, rough scallop, coquina
- I. auger, fighting conch, rough scallop, coquina

6. What is the mean absolute deviation of the data shown in the line plot, rounded to the nearest tenth? (6.SP.5c)



- A. 1.4
- B. 3
- C. 3.2
- D. 5

7. A family wants to buy tickets to a theme park. There are separate ticket prices for adults and children.

**Rollercoaster World!**

**Tickets:** \$30 for adults  
\$20 for children

Which expression represents the total cost, in dollars, for  $a$  adult tickets and  $c$  child tickets? (6.EE.2a)

- F.  $600(a + c)$
- G.  $50(a \times c)$
- H.  $30a + 20c$
- I.  $30a \times 20c$



8. What is the value of the expression below? (6.NS.3)



$$52.8 \div 0.16$$

9. What is the value of the expression below when  $a = 6$  and  $b = 14$ ? (6.EE.2c)

$$0.8a + 0.02b$$

- A. 0.4828                                      C. 5.08  
B. 0.8814                                      D. 16.4

10. Which property was *not* used in the box below to simplify the expression? (6.EE.3)

$$\begin{aligned} 0.3 \times 53 + 53 \times 0.7 &= 53 \times 0.3 + 53 \times 0.7 \\ &= 53 \times (0.3 + 0.7) \\ &= 53 \times 1 \\ &= 53 \end{aligned}$$

- F. Distributive Property  
G. Associative Property of Addition  
H. Identity Property of Multiplication  
I. Commutative Property of Multiplication

11. Determine a data set of 5 numbers that has the following measures:



- a mean of 7 and
- a median of 9.

Explain how you determined your data set. Then demonstrate that the mean of your data set is 7 and the median is 9. (6.SP.5c)

12. What is the value of the expression below? (6.RP.3c)

$$25\% \text{ of } 400$$

- A. 16    C. 1000  
B. 100    D. 10,000